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A simple model to calculate the Sherwood and Nusselt numbers for discs of various shapes

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Abstract—A simple model is presented for calculation of the Sherwood number and Nusselt number for discs of various shapes. For an atmometer of circular shape, experiments have been carried out in a low-speed wind tunnel in order to check the proposed model. The verification of the Sherwood and Nusselt numbers has been executed for Reynolds numbers ranging from 1.7×10^3 to 4.8×10^4. The model results for the Sherwood number are slightly underestimated and are within 6% in agreement with the experimental data. The model results for the Nusselt number are also underestimated, by about 8%.

1. INTRODUCTION

IN MICROMETEOROLOGICAL practices there is an increasing need to estimate the rate of the evaporation of bare soils and all kinds of vegetated areas. Most natural areas are not homogeneous but are irregular or disturbed by obstacles such as trees, shrubs, houses, etc. In order to obtain a good insight into the rate of evaporation of a complex terrain, evaporation measurements have to be carried out at a great number of locations. Practical realization of such experiments is very expensive and that is why there is an increasing need for simple and cheap measurement techniques.

The so-called atmometers are evaporation instruments which are small, hence easy to transport, simply constructed, hence easy to build, and, in addition, relatively cheap. These instruments normally consist of a wet blotting paper and a water reservoir. An example of such an instrument, which is widely used in micrometeorological practice, is the Piche atmometer [1].

Recently, a new atmometer, based on a capillary principle, has been developed [2] with fast-response characteristics. The instrument consists of a nickel-plated brass disc on which standard laboratory filter paper is placed. In the centre of the disc is a hole, which continues on through a capillary tube (see Fig. 1). By measuring the time needed for the water meniscus to rise 100 mm, the evaporation rate of the instrument can be estimated. Outdoor experiments have been carried out with this capillary atmometer in order to test its physical properties and practical usefulness [2, 3]. Moreover, in order to obtain a better understanding of the working mechanism of the instrument, the readings were compared with model calculations [3], based on the energy budget technique.

Some discrepancies emerged between the readings and the model calculations which could only be ascribed to uncertainties in the assumed Nusselt and Sherwood numbers. The assumed Nusselt and Sherwood numbers were those mentioned in Monteith [4] for rectangular and circular discs. Here, $Nu = 0.666Re^{1/2}\,Pr^{0.33}$ and $0.037\,Re^{0.8}\,Pr^{0.33}$ for the laminar and turbulent cases, respectively, and $Sh = Nu\,Le^{0.33}$. In fact, these are the well-known relations for flow parallel to a plane surface of finite length (see for example Jakob [5] and Schlichting [6]).

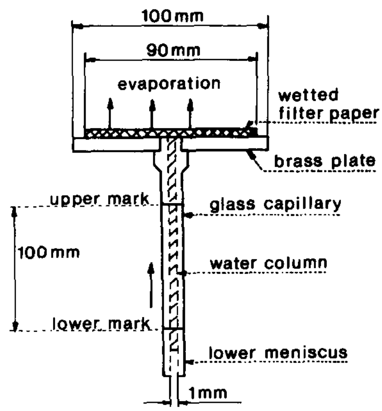

FIG. 1. Design of the capillary atmometer.

The Nusselt number and the Sherwood number given by Monteith [4] are those for flat plates and are not correct for circular discs. That is why it was decided, firstly, to measure accurately these numbers for the configuration of the capillary atmometer in a wind tunnel experiment. The experimental lay-out is given in Section 2.

In meteorological practice, atmometers of various configurations are used. That is why, secondly, a simple model has been constructed to calculate the Sherwood number and the Nusselt number for discs of any given shape. This model is explained in Section 3.

Thirdly, the model has been applied to the capillary

NOMENCLATURE

A_{12} ratio, surface filter paper and brass disc [—]
ΔC $C_a - C_d$ [—]
D vapour diffusivity [$m^2 s^{-1}$]
E evaporation [$kg m^{-2} s^{-1}$]
Gu Guhmann number $(T_a - T_d)/T_a$ [—]
L latent heat of evaporation [$J kg^{-1}$]
L_0 $(Y_m - Y_f)/2$ [m]
$\dot{m}$ mass flux [$kg m^{-2} s^{-1}$]
Nu Nusselt number, flat plate [—]
Nu_c Nusselt number, circular disc [—]
Pr Prandtl number [—]
$\dot{q}$ sensible heat flux [$W m^{-2}$]
R_d radius brass disc [m]
R_f radius filter paper [m]
Re Reynolds number [—]
Sc Schmidt number [—]
Sh Sherwood number, rectangular plate [—]
Sh_c Sherwood number, circular disc [—]
T_a temperature mean flow [K]
T_d temperature, disc [K]
ΔT $T_a - T_d$ [K]
V mean wind speed [$m s^{-1}$]
x, y coordinates [m]
Y effective length scale [m]
Y_d effective length scale brass disc [m]
Y_f effective length scale for mass transfer filter paper [m].

Greek symbols
ε emission coefficient [—]
λ heat conductivity [$W m^{-1} K^{-1}$]
ν kinematic viscosity [$m^2 s^{-1}$]
σ Stefan–Boltzmann constant [$W K^{-4} m^{-2}$].

Subscripts
a ambient
br brass
c circular
cr critical
d disc
f filter
l laminar
m momentum
pa paper
t turbulent.

atmometer and the calculations have been compared with the results obtained from the wind tunnel experiment. The results obtained are discussed in Section 4.

2. EXPERIMENTAL LAY-OUT

The nickel-plated brass disc of the atmometer had a diameter of 100 mm and a thickness of 2 mm. Before each measurement, the disc as well as the capillary was thoroughly cleaned with alcohol. As filter paper, standard laboratory filter paper from Whatman No. 4 was used. This filter paper was chosen because it has a fast infiltration speed (100 ml in 12 s). In the original design [2], the filter paper had a diameter of 90 mm. In the wind tunnel experiments, we used this diameter and, besides, we did some additional experiments with filter paper with a diameter of 55 mm.

Experiments were carried out in a low-speed wind tunnel. The cross-section of the 0.4 m long test chamber was rectangular (0.4 m × 0.4 m). In the test chamber the turbulent intensity was lower than 2%. In the wind tunnel, experiments were carried out with wind speeds ranging from 0.2 to 6.0 $m s^{-1}$. Lower and higher wind speeds were not possible due to inaccuracies of the wind speed measurements outside this speed range.

The temperature and the wet bulb temperature of the air flow were measured with an Assmann psychrometer. Moreover, at two locations on the disc, one at the top and one at the bottom, the temperature of the brass plate was measured with thin welded thermocouples. The welded thermocouples had a thickness of 0.02 mm. The top temperature was measured between the filter paper and the brass disc at about 5 mm downstream of the filter paper. The bottom temperature was measured at the farthest point downstream of the brass disc. The temperature of the disc was estimated at two different locations in order to check its homogeneity. It appeared that during the experiments both temperatures were equal to within 0.05 K, which meant within the experimental error.

The capillary of the atmometer had an inner diameter of 1 mm. The length of time needed for the meniscus to rise 100 mm was measured with a stopwatch.

The Sherwood number can be easily calculated and is, according to its definition

$$Sh = \dot{m} \cdot Y_f / (D \cdot \Delta C) \qquad (1)$$

where, Y_f is an effective mean length scale for mass transfer of the filter paper. The assessment of this length scale will be explained in the next section.

The Nusselt number has been estimated in an indirect way by using the energy budget technique. From the energy budget of the whole disc, it can be easily derived that the convection heat flux from the ambient air flow to the disc equals

$$\dot{q} = A_{12} L\dot{m} - \bar{\varepsilon}\sigma(T_a^4 - T_d^4). \qquad (2)$$

Here, $\bar{\varepsilon}$ is the mean emission coefficient of the disc

$$\bar{\varepsilon} = ((1 + (1 - A_{12})) \cdot \varepsilon_{br} + A_{12} \cdot \varepsilon_{pa})/2 \qquad (3)$$

where the emission coefficients of the brass disc, ε_{br}, and the wet filter paper, ε_{pa}, were taken to be 0.70 and

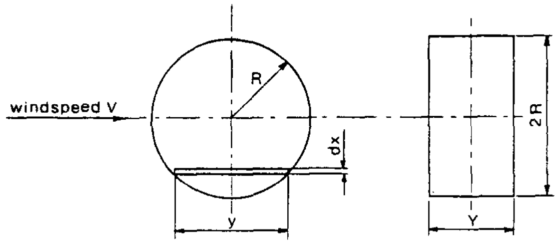

FIG. 2. Schematic outline of the transformation procedure.

0.97, respectively [7]. Next, the Nusselt number can be estimated:

$$Nu = (\dot{q} Y_d)/(\lambda \cdot \Delta T) \quad (4)$$

where Y_d is an effective mean length scale for heat transfer of the brass disc. The assessment of this length scale will be explained in the next section.

3. MATHEMATICAL MODEL

The proposed model to calculate the Sherwood number for circular discs is essentially based on the results of this number for flow parallel to a plane surface of finite length. To apply these results to a circular disc, the following procedure is executed. The disc is subdivided into small strips parallel to the mean flow with width dx and length y (left-hand side of Fig. 2). It is assumed that the mass transfer at a rectangular plane surface can be applied to these strips separately. The total mass transfer of the disc is obtained by integration over the whole width of the disc. In order to estimate a mean effective length scale, Y, for mass transfer of a circular disc, the total mass transfer of the disc is equalized to that for a rectangular disc with the same width as the circular disc and with length Y (right-hand side of Fig. 2). In this calculation, the same technique has been applied in estimating effective length scales as originally was proposed by Parkhurst *et al.* [8] for heat transfer at artificial leaves.

Smolsky and Sergeyev [9] intensively studied mass and heat transfer from free and porous surfaces into a turbulent air stream. For the Sherwood number they found that, for flat rectangular surfaces

$$Sh = f_t(Re, Sc, Gu) = 0.094 Re^{0.8} Sc^{0.33} Gu^{0.2} \text{ (turbulent case).} \quad (5a)$$

In the present paper, this result has been adopted for turbulent flow. For laminar flow the well-known relation [6]

$$Sh = f_l(Re, Sc) = 0.664 Re^{1/2} Sc^{0.33} \text{ (laminar case)} \quad (5b)$$

has been used.

Generally (see e.g. equation (5)), the Sherwood number can be expressed as a function of Re according to $A \cdot Re^n$. For the filter paper of the atmometer with radius R_f, an effective mean length scale, Y_f, can be defined by writing the total rate of evaporation from the filter paper as (see Fig. 3)

$$E = A \cdot (V \cdot Y_f/\nu)^n \cdot D \int_{-R_f}^{R_f} y \, dx. \quad (6)$$

The total rate of evaporation can also be written in the form

$$E = A \int_{-R_f}^{R_f} (Vy/\nu)^n D y \, dx. \quad (7)$$

By equating expressions (6) and (7), the mean effective length scale for mass transfer of the filter paper, Y_f, is

$$Y_f = \left(\int_{-R_f}^{R_f} y^n \, dx \Big/ \int_{-R_f}^{R_f} y \, dx \right)^{1/(n-1)} \quad (8)$$

where $n = 0.5$ in the laminar case and $n = 0.8$ in the turbulent case.

In the design of the atmometer, the diameter of the filter paper is smaller than the diameter of the brass disc (see Fig. 3). Consequently, the velocity boundary layer starts from the leading edge and grows progressively, while the vapour boundary layer starts further downstream, so that the boundary layers have different thicknesses. Physically, this means an additional resistance for the vapour transport. For flat plates, analytical expressions have been derived in order to correct for this additional resistance. The following corrections have been adopted as derived in [10]:

$$C_l = (1-(L_0/Y_m)^{0.75})^{0.67}/(1-L_0/Y_m) \quad \text{(laminar case)} \quad (9)$$

$$C_t = (1-(L_0/Y_m)^{0.98})^{0.82}/(1-L_0/Y_m) \quad \text{(turbulent case)} \quad (10)$$

where L_0 is the difference between the effective leading edge of the velocity boundary layer and the effective starting point of the vapour boundary layer as indicated on the right-hand side of Fig. 3, and Y_m is an effective length scale for momentum transport.

In order to find L_0 and Y_m, the mean effective length scale for momentum transfer at the brass disc of the capillary atmometer has to be estimated. Here, the same procedure as described above has to be carried out. If the general expression $B \cdot Re^m$ [9] is taken for the drag coefficient, the mean effective length scale for momentum transfer, Y_m, is

$$Y_m = \left(\int_{-R_d}^{R_d} y \, dx \Big/ \int_{-R_d}^{R_d} y^m \, dx \right)^{1/m} \quad (11)$$

where $m = 0.5$ in the laminar case and $m = 0.2$ in the turbulent case. Because only the surface of the disc which has flow trajectories which coincide with the filter paper is of importance, the integration must be carried out over the width of the filter paper only.

Finally, if it is assumed that the flow is laminar for

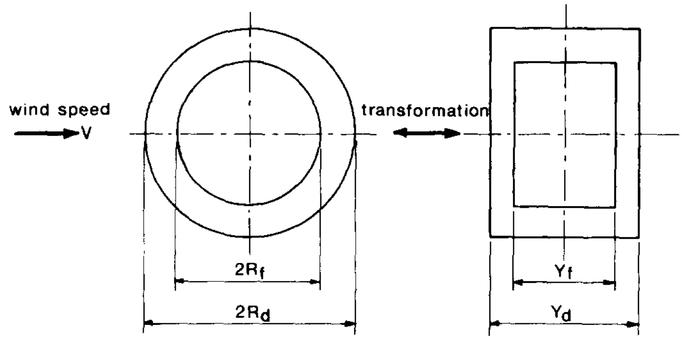

FIG. 3. Transformation of the circular disc–filter paper system into a rectangle.

Re lower than the critical Reynolds number, Re_{cr}, of 20 000 [6], the mathematical expression for the Sherwood number of the circular disc of the capillary atmometer, Sh_c, is

$$Sh_c = f_l(Re_{cr}, Sc)\cdot C_l + (f_t(Re, Sc, Gu) - f_t(Re_{cr}, Sc, Gu))\cdot C_t. \quad (12)$$

In order to estimate the Nusselt number for the circular disc of the capillary atmometer, the same procedure as described above can be executed. The vapour concentration and the temperature are both scalar quantities and the transport processes of heat and mass behave more or less similarly. Consequently, the solution to any problem in forced convection mass transfer may be made to provide a solution to a corresponding problem in heat transfer, by putting it into the form of equation (5) and then replacing the dimensionless groups *Sh* and *Sc* by their counterparts *Nu* and *Pr*. This procedure may be applied to any equation obtained empirically [6].

For heat transfer into a turbulent air flow, we use the Nusselt numbers for flat plates as given by Smolsky and Sergeyev [9]:

$$Nu = g_t(Re, Pr, Gu) = 0.086 Re^{0.8}\, Pr^{0.33}\, Gu^{0.2} \text{ (turbulent case)} \quad (13a)$$

and for the laminar state we use the following well-known results [6]:

$$Nu = g_l(Re, Pr) = 0.664 Re^{1/2}\, Pr^{0.33} \text{ (laminar case).} \quad (13b)$$

For the mean effective length scale, Y_d, for heat transfer at a circular disc, the same expression as equation (8) will be found. In the configuration of the capillary atmometer, heat transfer occurs over the whole disc. That is why, in estimating Y_d, the integration in equation (8) must be carried out over the whole width, $2R_d$, of the brass disc.

In the applied configuration of the capillary atmometer, the velocity boundary layer and the temperature boundary layer both start at the same leading edge. Consequently, no corrections, due to differences in boundary layer thicknesses, need to be carried out. Finally, the Nusselt number of the capillary atmometer, Nu_c, is found to be

$$Nu_c = g_l(Re_{cr}, Pr) + g_t(Re, Pr, Gu) - g_t(Re_{cr}, Pr, Gu). \quad (14)$$

4. RESULTS AND DISCUSSION

The measurement results of the Sherwood number, *Sh*, as well as the model results, Sh_c, have been depicted in Fig. 4 as a function of *Re*. The *Re* has been calculated with the same characteristic length scale, Y_f, as has been used in Sh_c. In Fig. 5, the same result for the 55-mm filter paper has been depicted. Moreover, to gain a better idea of the differences between model results and measurement data, in Figs. 6 and 7 both results have been compared.

From these results it can be concluded that, roughly speaking, the model fits the experimental results well. At a closer look, however, the model underestimates the experimental data for the ratio disc–filter paper of 100/90. If the result of Fig. 6 is fitted with a linear regression it appears that the model results are shifted down by about 6% in the given Reynolds number range.

A small underestimation in the model results must be expected, however, because in the model the influence of the side edges effect on the mass exchange has been ignored. Besides, it must also be expected that the side edge effect will cause serious discrepancies if the model is applied to elliptical discs; the greater the aspect ratio the greater the discrepancy.

The results for *Nu* versus *Re* are given in Figs. 8 and 9. The *Re* has been calculated with the same characteristic length scale, Y_d, as has been used for *Nu*. The model results and measurement data are compared in Figs. 10 and 11.

At first sight, it is seen from the results for the Nusselt number that the model fits the experimental data less well than in the case for the Sherwood

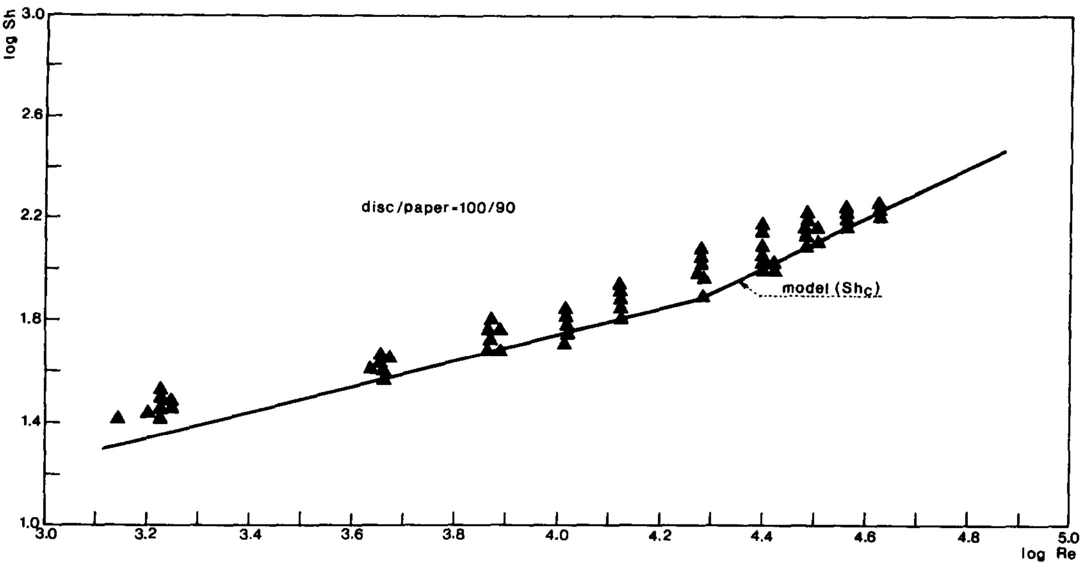

FIG. 4. Sh_c vs Re for ratio disc–filter paper of 100/90.

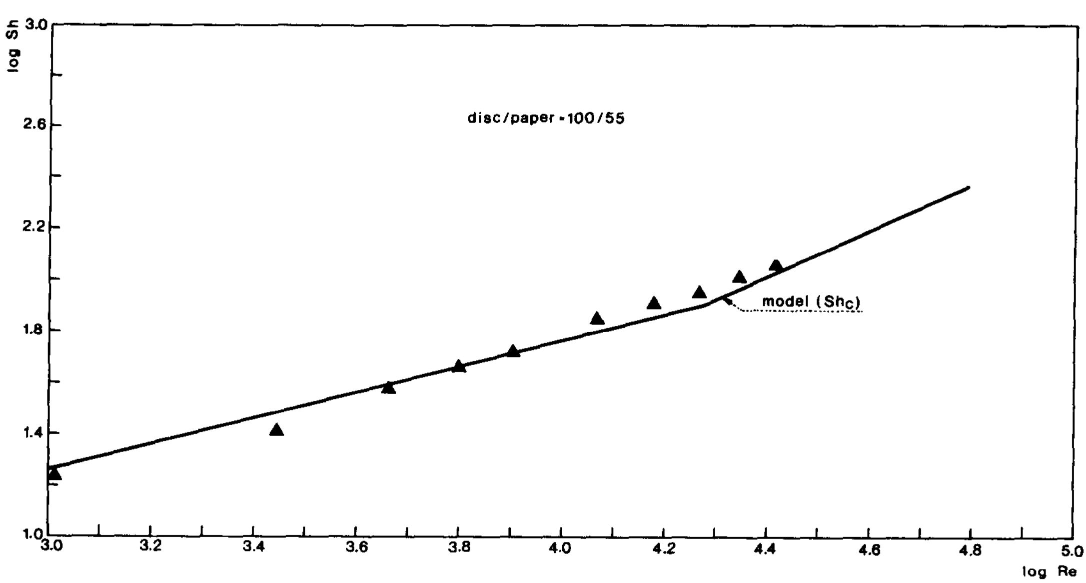

FIG. 5. Sh_c vs Re for ratio disc–filter paper of 100/55.

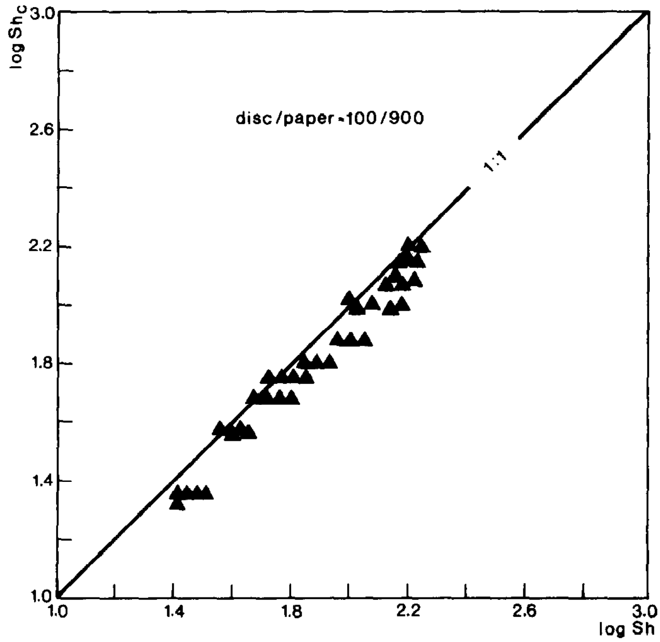

FIG. 6. Comparison model results with the experimental data for a ratio disc–filter paper of 100/90.

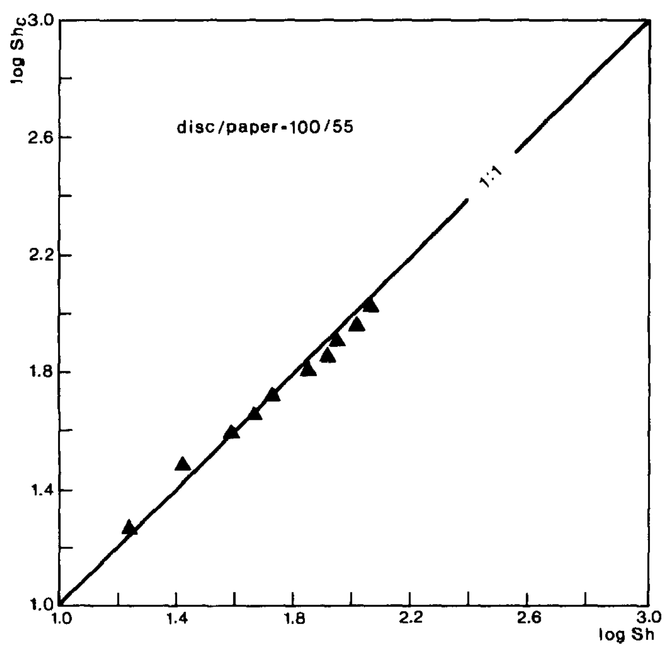

FIG. 7. Comparison model results with the experimental data for a ratio disc–filter paper of 100/55.

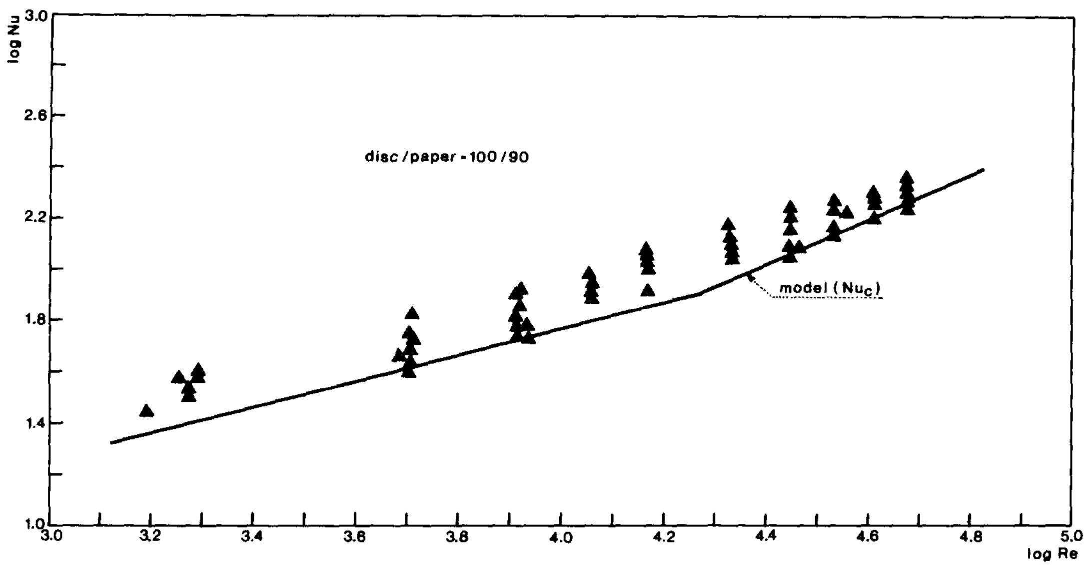

FIG. 8. Nu_c vs Re for ratio disc–filter paper of 100/90.

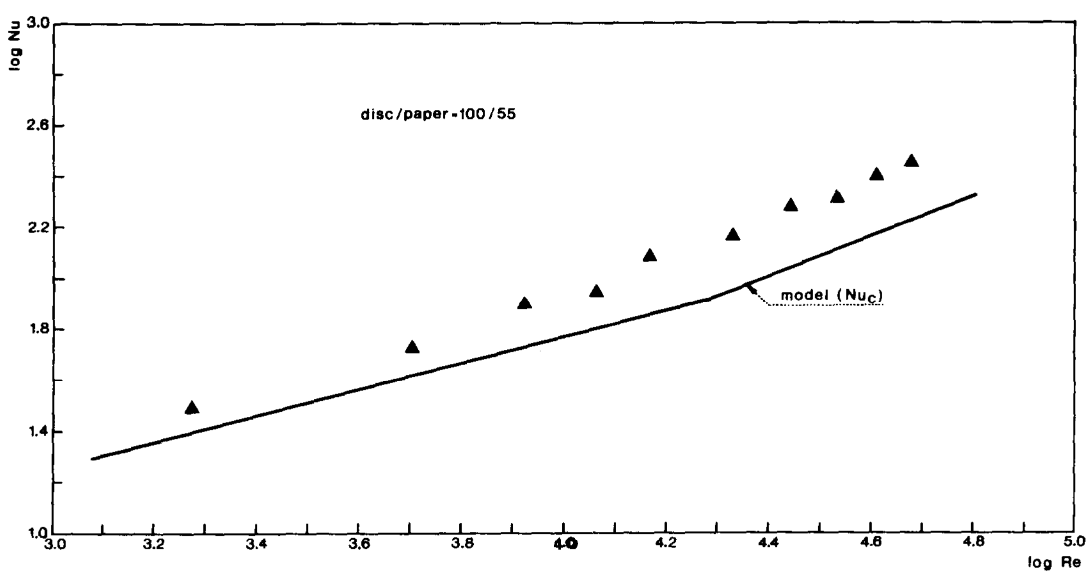

FIG. 9. Nu_c vs Re for ratio disc–filter paper of 100/55.

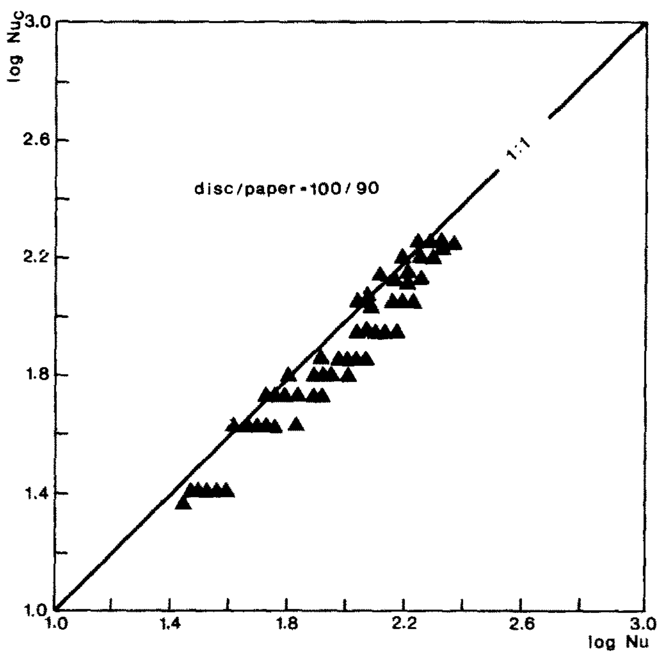

FIG. 10. Comparison model results with the experimental data for a ratio disc–filter paper of 100/90.

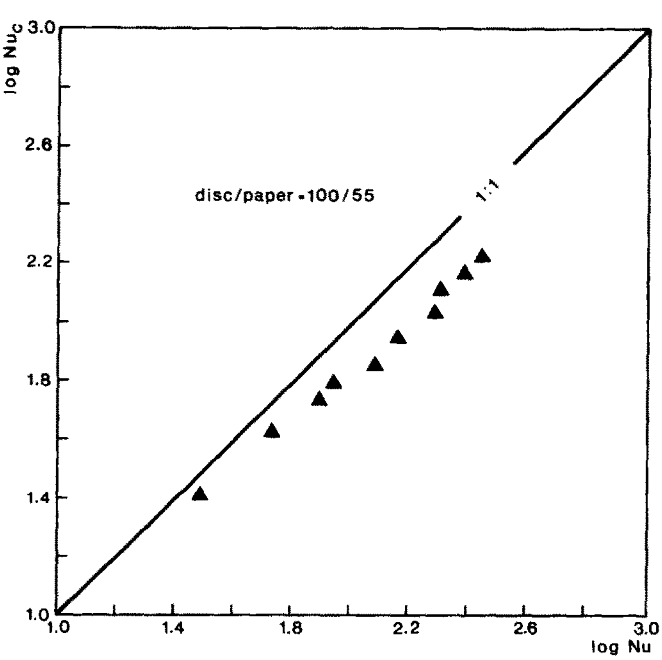

FIG. 11. Comparison model results with the experimental data for a ratio disc–filter paper of 100/55.

number. If both results of Figs. 10 and 11 are fitted with a linear regression, it appears that the model results are shifted down by about 8% in the given Reynolds number range.

For the same reason as mentioned before, an underestimation of the model results must be expected here. Besides, two additional reasons can be given for explaining the bigger discrepancy in *Nu* results.

Firstly, the capillary connected at the bottom of the plate influences the air flow. That means that the real heat exchange at the bottom of the plate is somewhat better than at the top due to wake effects behind the capillary; the capillary affects the heat exchange at the bottom of the disc and not the mass transport since mass transport occurs at the top of the disc only. Consequently, the model results will be slightly underestimated.

Secondly, the Nusselt number has been calculated in an indirect way by applying the heat budget at the plate. Here, the convection heat flux has been calculated as the remainder term of the latent heat and the net radiation term. Consequently, all uncertainties will emerge in the final experimental results of *Nu*.

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UN MODELE SIMPLE DE CALCULER LE NOMBRE DE SHERWOOD ET NUSSELT POUR LES DISQUES DE CONFIGURATION DIFFERENTS

Résumé—Un modèle simple est présenté de calculer de nombre de Sherwood et de Nusselt pour des disques de configuration différents. Pour un disque circulair d'un atmomètre, des expériences sont conduites dans un soufflerie de champs pour controler le modèle présente. La verification des nombres de Sherwood et Nusselt est conduite pour les nombres de Reynolds entre $1{,}7 \times 10^3$ et $4{,}8 \times 10^4$. Les résultats modèle des nombres de Sherwood sont sous-estimés et correspondent un peu près a 6% avec les resultats expérimentelles. Les résultats modèle des nombres de Nusselt sont sousestimés aussi et correspondent un peu près a 8% avec les résultats experimentelles.

EIN EINFACHES MODELL ZUR BERECHNUNG DER SHERWOOD UND NUSSELTZAHL FUR SCHEIBEN VERSCHIEDENER FORM

Zusammenfassung—Ein einfaches Modell zur Berechnung der Sherwood und Nusseltzahl für Scheiben verschiedener Form wird präsentiert. Für einen Atmomesser mit runden Form sind Experimenten ausgeführt worden in einem Windkanal um das vorgeschlagen Modell nach zu prufen. Die Verifikation der Sherwood und Nusseltzahlen ist im Reynoldszahlbereich zwischen $1{,}7 \times 10^3$ und $4{,}8 \times 10^4$ ausgeführt worde. Die Modellergebnisse für die Sherwoodzahlen werden ungefähr mit 6% unterschätzt in Vergleich mit den Experimenten. Die Modellergebnisse für die Nusseltzahlen werden in Vergleich mit den Experimenten mit ungefähr 8% unterschätzt.

ПРОСТАЯ МОДЕЛЬ ДЛЯ РАСЧЕТА ЧИСЕЛ ШЕРВУДА И НУССЕЛЬТА ДЛЯ ДИСКОВ РАЗЛИЧНОЙ ФОРМЫ

Аннотация—Предложена простая модель для расчета чисел Шервуда и Нуссельта для дисков различной формы. Модель проверялась в экспериментах с эвапорометром круглой формы в аэродинамической трубе малых скоростей. Проверка чисел Шервуда и Нуссельта проводилась при значениях числа Рейнольдса в диапазоне $1{,}7 \cdot 10^3$–$4{,}8 \cdot 10^4$. Расчетные значения числа Шервуда оказались несколько заниженными и совпадали с экспериментальными данными с погрешностью в 6%. Расчетные значения числа Нуссельта также оказались заниженными примерно на 8%.